Math 10B with Professor Stankova
Quiz 4; Tuesday, 2/13/2018
Section #211; Time: 11 AM
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Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True **FALSE** If we want to show that the statements S_n are true for all $n \geq 0$, we need to prove the base case $n = 1$.

Solution: The base case is $n = 0$.

2. **TRUE** False When $A \subset B$, the conditional probability $P(A|B)$ can be expressed as the fraction $\frac{P(A)}{P(B)}$ (given all involved quantities are well-defined).

Solution: Since $A \subset B$, we know that $A \cap B = A$ and hence

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}.$$

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (4 points) Prove that $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$ for all $n \geq 1$.

Solution: First we prove the base case $n = 1$. Then the LHS is 1 and the RHS is $\frac{1(1+1)}{2} = 1 =$LHS as required.
Now assume the inductive hypothesis IH: $1+2+\cdots+n = \frac{n(n+1)}{2}$ for some $n \geq 1$.
Now we want to prove that $1 + 2 + \cdots + (n+1) = \frac{(n+1)(n+2)}{2}$. We have that the left hand side is

$$LHS = (1+2+\cdots+n)+(n+1) \overset{IH}{=} \frac{n(n+1)}{2}+n+1 = \frac{n^2+n+2n+2}{2} = \frac{n^2+3n+2}{2}.$$

And

$$RHS = \frac{(n+1)(n+2)}{2} = \frac{n^2+3n+2}{2} = LHS.$$

Finally, by MMI, we know that $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$ for all $n \geq 1$.

(b) (3 points) What is the probability that when picking a hand of 5 cards out of a deck of 52 cards, you pick at least one king?

Solution: We can solve this via complementary probability. We have that $P(\geq 1K) = 1 - P(< 1K) = 1 - P(0K)$ and $P(0K) = \frac{\binom{48}{5}}{\binom{52}{5}}$. Thus, we have that

$$P(\geq 1King) = 1 - \frac{\binom{48}{5}}{\binom{52}{5}}.$$

(c) (3 points) What is the probability that when picking a hand of 5 cards out of a deck of 52 cards, you pick exactly three kings given that you have at least one king?

Solution: We can calculate the probability of picking three kings. The number of ways of picking three kings is $\binom{4}{3} \cdot \binom{48}{2}$ and the total number of ways is $\binom{52}{5}$. Thus, the conditional probability is

$$P(3K| \geq 1K) = \frac{P(3K \cap \geq 1K)}{P(\geq 1K)} = \frac{P(3K)}{P(\geq 1K)} = \frac{\frac{\binom{4}{3}}{\binom{48}{2}}}{1 - \frac{\binom{48}{5}}{\binom{52}{5}}}$$

$$= \frac{\binom{4}{3}\binom{48}{2}}{\binom{52}{5} - \binom{48}{5}}.$$